Another Traceability Path for Capacitance Measurements

by Henry P. Hall

The National Bureau of Standards recently announced new services for the rf calibration of impedance standards fitted with precision 14-mm coaxial connectors, like several manufactured by GR.[1,2,3] High frequency bridges and twin-T circuits developed at the Bureau are used for the measurement of resistance, capacitance, and inductance at listed frequencies of 0.1, 1.0, and 10 MHz.[4,5] Anyone who wants direct NBS "traceability" of these standards may send them to NBS for calibration at these test frequencies. This is a significant advancement in the measurement art.

The new service, however, raises an important question: "When are these *direct NBS calibrations* necessary?" This can be considered from two points of view: "When are these calibrations required to obtain a specific accuracy with high confidence?" and "When are they required to prove traceability as defined (somewhat loosely) by the DOD?" The two points of view diverge by varying degrees, unfortunately, particularly when we consider to whom we are trying to prove traceability. We think that the question can be answered satisfactorily for rf-capacitance calibrations, if you are interested in obtaining a *given required accuracy* through use of frequency-correction data. We hope that reason will prevail among metrologists and that the same criteria used in deciding what is sufficiently accurate can also be used to show traceability, for the benefit of quality-assurance inspectors.

The frequency corrections described below were used with apparent acceptance[6] before the new services were available. They still must be used at frequencies between those for which calibrations are now routinely provided.

Actually, all the new NBS rf-impedance calibrations, R, L, and C, are based on the frequency characteristics of air capacitors. While air capacitors vary with frequency in a very simple manner (see below), resistors and inductors vary with frequency in too complex a manner to be as predictably accurate in the rf range. Therefore, these new NBS bridges and twin-T's relate L and R back to fixed capacitors and capacitance differences in variable air capacitors.[7,8]

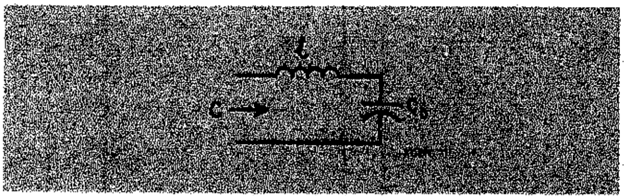

Figure 1. Simple equivalent circuit for two-terminal air capacitor.

RF Characteristics of Air Capacitors

The simple equivalent circuit of Figure 1 is surprisingly good for representing the effective capacitance of a two-terminal air capacitor. The effective capacitance is

$$C = \frac{C_o}{1 - \omega^2 l C_o} \qquad (1)$$

where C_o is the low-frequency value of capacitance. This equivalent circuit does not include series resistance, which does not appreciably affect the capacitance (even the parallel capacitance) if it is reasonably small. Neither does it include the effects of distributed inductance and capacitance. Also, at high frequencies the skin depth causes a slight reduction in inductance. These effects can be considered as variations in the value of l, with C_o being considered a constant (as long as all measurements are made at low humidity, $<40\%$ RH).

The simplest method of establishing l is to use a grid-dip meter to determine the resonant frequency of the shorted capacitor. This method, together with the above formula, was recommended for many years by NBS[6] to make rf-capacitor calibrations. For capacitors fitted with binding posts or banana-pin terminals, this is very easy to do, although we have to decide how much of the total inductance is in the shorting connection. When coaxial connectors are used, we remove the case to allow coupling to the meter coil.

The value of l obtained from a resonance measurement differs slightly from a lower-frequency value mainly because of the distributed parameters. The worst case would be that of a uniform line whose effective capacitance could be written:

$$C = \frac{C_o}{\left[1 - \omega^2 l C_o - \frac{\omega^4 l^2 C_o{}^2}{5} - \frac{2\omega^6 l^3 C_o{}^3}{35} \ldots\right]} \qquad (2)$$

The derivation is a power series in increasing powers of $\omega l C_o$. The first term is the important one for corrections at frequencies well below resonance. When $\omega^2 l C_o = 10\%$ the next term is 0.2%, and each succeeding term is much smaller still. This effective l is 1/3 the value of the total inductance. If C_o is assumed constant, the effective inductance determined by a resonance measurement would be higher than l by the factor $\frac{12}{\pi^2} = 1.21$ or 21%.

In large air capacitors (50 pF to 1000 pF) with parallel-plate construction,[1] most of the inductance is in the coaxial connector and its connection to the "stack" of plates. Most of the capacitance is in the stack itself so that the lumped equivalent circuit is a good approximation. The slight difference between the resonant value of l and the desired correction value is small, less than the uncertainty of the measurement. Many measurements were made at General Radio to establish this difference so that a more accurate value of l could be determined.[9] The values obtained agreed closely with NBS determinations (within 10%).

The smaller capacitors (1 to 20 pF) have an appreciable distributed capacitance; consequently, the value of resonant inductance will be in error, but by less than 21%. The effective value for these units was determined by a slotted-line technique.[9] These values, supplied by General Radio for our line of coaxial standard capacitors, are not for use above a specified frequency (250 MHz for the 20- and 10-pF units and 500 MHz for the smaller units). This restriction is mainly because of the variations due to distributed parameters. (Correction curves are supplied with units at higher frequencies.)

An important point, which is verified by the many measurements described above, is that different capacitors of the same value and construction have equal inductance, well within the accuracy of the determinations. This is not surprising. The inductance is mostly in the coaxial line of the

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precision connector, or in the stamped metal pieces that make connections to the plates. Measured variations in l, between similar units, are generally less than 3%, and allowing for a 10% change would be very conservative.

When are Calibrations Necessary?

Equation (1) can be rewritten:

$$C = \frac{C_o}{1 - \omega^2(l_s \pm \Delta l)C_o} \qquad (3)$$

where l_s is the specified value of inductance and $\pm\Delta l$ the possible deviation in this value. This deviation is due to error in determination of the inductance and its changes with frequency or between units. The low-frequency value C_o can be easily determined to 0.01% ± 0.005 pF on a GR 1615-A Capacitance Bridge by use of the coaxial adaptor.[9] This value can be used as long as $\omega^2 l_s C_o \times 100\%$ is less than the accuracy required. Corrections should be used above this frequency, and may be used with confidence as long as $\omega^2 \Delta l C_o \times 100\%$ is less than the accuracy required. If greater accuracy is required an NBS calibration should be made. Remember, however, that NBS accuracy is based on similar measurement techniques, subject to similar uncertainties.

The question now is: what is the value for Δl? A value of $\Delta l = 0.2\ l_s$ can be used with confidence as long as the total correction is 10% or less, the maximum frequency for the low values of capacitance is not exceeded, and humidity is reasonably low. Typically, errors will be substantially less. We suggest that calibration be considered traceable to an uncertainty of $\omega^2 N l_s C_o \times 100\%$ where N is some reasonable fraction. If we agree that measurements are traceable, based on some very conservative value of N, even as high as 0.5, this approach is still very useful.

We wish to stress that it is the small *correction* which is important because C_o can be easily determined. The high-frequency capacitance value will vary with time, temperature, or shock because C_o changes with these effects. But unless there is catastrophic damage, the inductance will not change enough to be perceptible. Therefore, *differences* between high-frequency and low-frequency calibrations should be recorded and once made on a given unit need not be repeated. This point should be appreciated also by quality-assurance inspectors.

Conclusion

The principle of applying conservative tolerances to frequency corrections has relevance to frequency translation of other types of impedances. Capacitors with dielectric materials other than air have additional sources of deviations, but these can be determined within definite limits and included in the over-all uncertainty. Frequency corrections are significant for larger capacitors at much lower frequencies, and the corrections greatly extend the useful frequency range of the capacitors. This principle may be applied also to inductors and resistors, even though their equivalent circuits are more complex.

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*Reprints available from General Radio.

**Presented at the Conference on Precision Electromagnetic Measurements, Boulder, Colorado, June 2-5, 1970.

***Presented at the Annual Conference of the Precision Measurements Association, Washington, D. C., June 17-19, 1970.