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Frequency Measurement by Sliding Harmonics

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Frequency Measurement by Sliding Harmonics*

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Summary—A method of measuring radio frequencies is described which uses an interpolating, or adjustable, frequency standard. Harmonics of this standard are caused to slide along the frequency scale until the one next below the frequency under measurement is brought up to match that frequency. The number of the used harmonic is readily determined from a simple calibration of the detector, heterodyne frequency meter, or receiver used to receive the frequency being measured. The positive increment in frequency of the used harmonic is determined from the control dial of the adjustable frequency standard. The use of wide-band receivers and interpolating equipment is avoided. Accuracy of measurement of the order of 10 parts per million is realized with the equipment described. If the interpolating frequency standard is treated as a highly stable heterodyne frequency meter, it may be used with many advantages in a conventional frequency-measuring system. The methods discussed are applicable to frequencies up to 1000 Mc.

A WIDELY USED, and generally very satisfactory, method of measuring frequencies up to several megacycles utilizes (1) a series of fixed standard-frequency harmonics of 10 kc, (2) a receiver or detector capable of accepting the frequency to be measured as well as the nearest 10-kc standard-frequency harmonic, and (3) an interpolation oscillator covering a range from zero to one-half the standard frequency (0–5 kc) with good stability and a highly expanded scale.

This method is very satisfactory for measuring the frequency of local oscillators or of transmitters having steady carrier frequencies. In cases of keyed or intermittent signals, an auxiliary oscillator (heterodyne frequency meter) is frequently required as a substitute signal source. Finally, unless an auxiliary oscillator is used, the system is not capable of producing a desired output frequency.

If the frequency f_x to be measured lies in the interval between a given standard-frequency harmonic nf_s, and a frequency $f_s/2$ above that harmonic, the beat-frequency difference obtained in the output of the receiver is considered positive. The frequency under measurement is then given by $f_x = nf_s + f_{b1}$. If the frequency to be measured lies above the given standard-frequency harmonic nf_s by more than $f_s/2$ but less than f_s, attention is shifted to the next higher standard-frequency harmonic $(n+1)f_s$, and the beat-frequency difference is considered negative. The frequency under measurement is then given by $f_x = (n+1)f_s - f_{b2}$. These conditions are illustrated in Fig. 1. Operating in this manner, the range of the interpolation oscillator needs to be only from 0 to $f_s/2$.

If, now, we propose to measure frequencies of a few hundred megacycles by this method, we find (1) that the standard frequency must be increased by 100 times, say, in order that the necessary harmonics can be generated with usable intensity and that the separation between successive harmonics be sufficient for ready identification; (2) that the pass band of the detector or receiver must correspondingly be increased by 100 times;

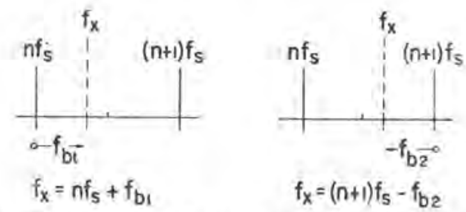

Fig. 1—Illustrating frequency measurement by conventional methods, with a fixed frequency standard.

and (3) that the range of the interpolation oscillator must also be increased by 100 times. It is apparent that the beat-frequency difference obtained in the output of the receiver will range far beyond audible limits, thereby requiring means of indication other than headphones or loudspeakers. With the extended range required of the interpolation oscillator, decreased accuracy of reading results. While it is by no means impossible to set up a frequency-measuring system on this basis, such a system is neither convenient or simple to use.

Some improvement is possible by utilizing the *higher* of the two beat frequencies produced by beating the frequency to be measured with adjacent standard-frequency harmonics and using an interpolation oscillator covering the range from $f_s/2$ to f_s (instead of from 0 to $f_s/2$), but the basic difficulties remain.[1]

A search for a more rapid procedure leads to the following: Consider a frequency standard combined with an interpolator so as to produce an *adjustable* standard frequency, of stability and accuracy approaching that of a fixed standard. It is then evident that a multiple of the adjustable standard frequency can be slid along the frequency scale and be matched to a frequency under measurement by simple zero-beating in any convenient detector or receiver. Under these conditions no wide-band receivers or wide-range interpolation oscillators are required.[2]

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[1] S. Sabaroff, "An ultra-high-frequency measuring assembly," PROC. I.R.E., vol. 27, pp. 208–213; March, 1939.

[2] J. K. Clapp, "Continuous interpolation methods," *General Radio Experimenter*, vols. 8 and 9, pp. 4–8 and 3–8; January and February, 1944.

The measurement of a frequency under these conditions is indicated in Fig. 2. With the control of the adjustable standard set at zero—that is, with true standard-frequency output—the unknown frequency lies above a given standard harmonic nf_s, as shown in line A. Now the standard frequency is altered, toward a *higher* frequency f_s', so that the given harmonic n slides toward higher frequencies and finally is matched against the unknown frequency f_x, as shown in line B. Knowing the harmonic number n, and the new value of standard frequency f_s', the unknown frequency is given by $f_x = nf_s'$.

For greater convenience, the altered value of standard frequency can be thought of as the unaltered value f_s *plus* an increment in frequency of $(f_s' - f_s)$. The frequency of the nth harmonic is, then,

$$nf_s' = nf_s + n(f_s' - f_s).$$

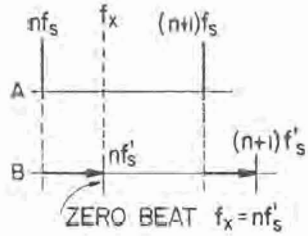

Fig. 2—Illustrating frequency measurement by use of an adjustable frequency standard.

To obtain complete coverage at harmonic n it must be possible to adjust the nth harmonic over the interval from nf_s to $(n+1) f_s$. Inspection shows that the maximum value of $n(f_s' - f_s)$ must be equal to f_s, or in fractional form, $n\Delta = 1$. Since the fractional change in frequency, in passing from any harmonic higher than n to the next harmonic above it, is less than that at harmonic n, it follows that complete coverage is obtained not only over the interval from n to $(n+1)$ *but also over any higher harmonic interval.*

On this basis, equipment has been designed especially for use with a heterodyne frequency meter covering 100 to 200 Mc with a dial calibrated at 1-Mc intervals. To interpolate between adjacent dial markings requires an interpolating standard of 1 Mc base frequency. At 100 Mc, n then equals 100 and, consequently, $\Delta = 0.01$. The fundamental range of the interpolating standard is then from 1.000 to 1.010 Mc. The standard was made up with a 950-kc crystal-controlled oscillator and a stable bridge-controlled variable-frequency oscillator of 50 to 60 kc. The sum of these two frequencies is then used as the output frequency. The operating control is a worm drive with a scale of 1000 divisions. A 1-Mc multivibrator controlled by the standard provides useful output at all harmonics from 100 to 200 or more.

A block diagram of the assembly is shown in Fig. 3. The output spectrum is indicated, where some one of the output harmonics is to be matched to the fundamental frequency of the heterodyne frequency meter.[3]

In operation, the heterodyne frequency meter is first set to zero beat with f_x. This may be done at the fundamental or at a harmonic of the frequency meter. (If at a harmonic, the number of the harmonic is determined by use of the heterodyne frequency meter.) The frequency meter is then left at this zero-beat setting. The harmonic output of the interpolating frequency standard is then injected into the heterodyne frequency meter and the standard control is advanced until the first loud beat tone is obtained; the control is then finally adjusted for zero beat. The number of the standard harmonic used in the measurement is determined from the heterodyne-frequency-meter reading. The number of interpolator divisions required to slide the used standard harmonic upward in frequency by just 1 Mc is given by a table. The amount, as a fraction of 1 Mc, that it has actually been moved is given by the ratio of the actual dial reading to the tabulated value. This amount is *added* to the number of the used harmonic to obtain the final result in megacycles.

A numerical example may be easier to follow. Suppose the reading of the heterodyne frequency meter at zero beat with f_x is 162.3 Mc. The interpolator control is advanced, say, 249.0 divisions to obtain zero beat against the frequency meter. The used harmonic is 162 (the next integral value below the reading of 162.3); entering the table for the range 162–163 Mc, it is found that the interpolator dial must be advanced by 617.3 divisions to cover the 162–163-Mc range. Since the dial was actually advanced 249.0 divisions, the fraction 249.0/617.3 of a megacycle was actually covered,

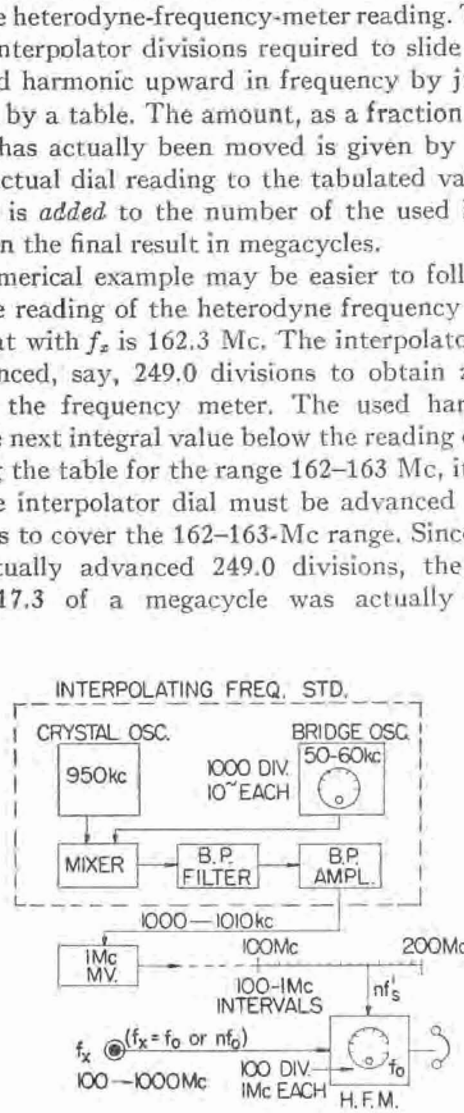

Fig. 3—A particular application of the interpolating frequency standard. Harmonic extension is used to expand the effective range.

[3] The equipment referred to comprises the General Radio Company Type 720 heterodyne frequency meter; Type 1110 interpolating frequency standard; and Type 1110-P1 multivibrator unit, containing 1-Mc and 0.1-Mc multivibrators.

amounting to 0.404 Mc. The final result is then 162 +0.404 Mc=162.404 Mc.

For purposes of illustration, the harmonic generator indicated in Fig. 3 is shown simply as giving multiples of the frequency of the interpolating frequency standard. Actually, one or more additional harmonic generators can be used; for example, a unit generating multiples of 0.1 Mc. Again, use is made of the multiples from 100 to 200, covering the range from 10 to 20 Mc in 0.1-Mc steps. A frequency meter covering 10 to 20 Mc and graduated in 0.1-Mc intervals would then be used exactly in the manner outlined above.

Since harmonic extension of the measurement range is readily accomplished by use of the frequency meters, the lower-frequency arrangement just described is useful to cover the range from 10 to 100 Mc, using no higher than the fifth harmonic of the frequency meter. Similarly, the higher-frequency arrangement first described is useful to cover the range from 100 to 1000 Mc, again using no higher than the fifth harmonic of the frequency meter. Both units then cover from 10 to 1000 Mc.

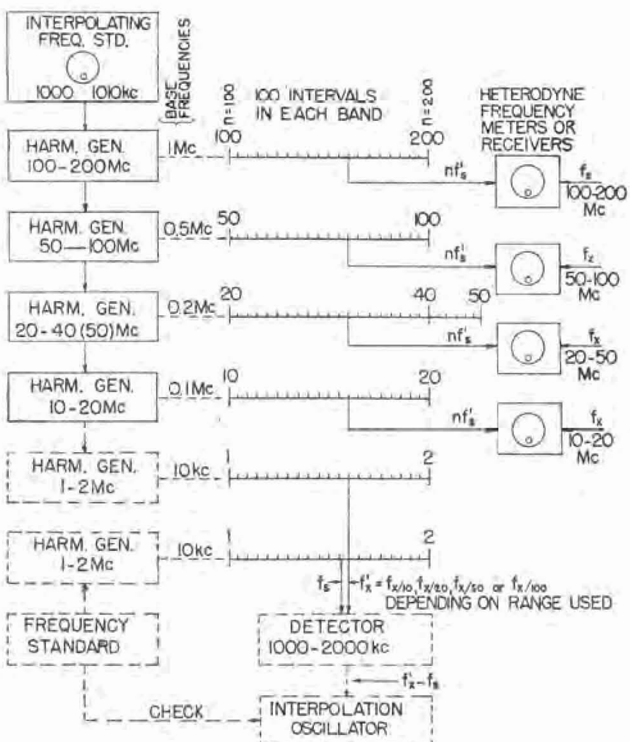

Fig. 4—An application of the interpolating frequency standard to measurement of frequency over a wide range without the use of harmonic extension (solid portion). The interpolating frequency standard can be used to advantage as a very stable heterodyne frequency meter in a conventional measuring system (dashed portion).

If it is desired to cover wider frequency ranges than 2:1 without the use of harmonic extension, a number of harmonic generators in cascade can be used, each with a suitable frequency meter or receiver (or a single multi-range instrument). The individual harmonic generators cover approximately 2:1 in frequency, and successive generators can be arranged on a 1, 2, 5, 10 basis as indicated in Fig. 4.

This arrangement covers an over-all range of 10 to 1 using three harmonic generators, and 20 to 1 using four. The use of multiples from 100 to 200, in the case of the 20- to 40-Mc unit, leaves a gap between the coverage of this unit and that of the next higher, 50 to 100 Mc. There is nothing to prevent the use of multiples *higher* than the 200th as required to extend the coverage from 40 to 50 Mc. If, however, the use of *lower* multiples is attempted, so as to obtain complete coverage from 40 to 100 Mc, it would be necessary to redesign the interpolating frequency standard for complete coverage over the range from 80 to 81 Mc (instead of 100 to 101 Mc, as described).

Extension to higher frequencies is possible by the arrangement shown in Fig. 5. Here the multivibrator type of multiplier is replaced by distorting amplifiers. The output frequency of the interpolating frequency standard is multiplied by 5, amplified, and then put through several multiplier stages. Outputs of all multiplier stages are fed to a crystal rectifier, the output of which provides multiples of 5 Mc in the range from 500 to 1000 Mc. Used with a heterodyne frequency meter of 500 to 1000 Mc,[4] the operation is exactly as described above.

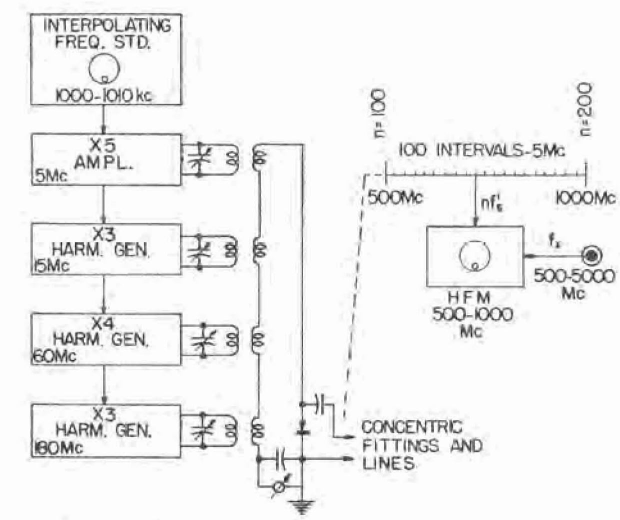

Fig. 5—A method of extending the range of measurement of a particular interpolating frequency standard.

It is evident that the accuracy of measurement depends on three principal factors: (1) The accuracy with which the adjustable frequency can be matched to the frequency under measurement; (2) the accuracy with which the increment in frequency can be determined

[4] General Radio Company Type 1110-P2 harmonic multiplier and Type 1021-A heterodyne frequency meter; both under development.

from the interpolation dial readings; and (3) the accuracy of the crystal-controlled oscillator.

In general, if the stability of the frequency under measurement is good, the matching can be carried out with an accuracy which does not limit the over-all accuracy of measurement. The accuracy of determining the frequency increment is limited by the linearity of scale of the interpolation oscillator and by drift, if means are not provided for correction. This limitation is of the order of ± 30 parts per million. However, if a detector and audio-frequency amplifier are provided, it is possible to check and to correct, if necessary, the calibration of the interpolation oscillator in terms of the crystal-controlled oscillator. Such checks can be obtained at a large number of points over the scale of the interpolation oscillator. Properly correcting the interpolator oscillator by this means reduces the interpolator scale error to the order of ± 2.5 parts per million, *in terms of the crystal-controlled oscillator.* In the final analysis, the over-all accuracy is limited by the accuracy of the crystal-controlled oscillator, which, with the simple arrangement used here, is about ± 5 parts per million. By checking the crystal oscillator against standard-frequency transmissions, or an accurate frequency standard, from time to time, and correcting, if necessary, this error can be held within smaller limits. As a final figure, it is estimated that the over-all accuracy which can be readily realized is of the order of 10 parts per million (0.001 per cent).

Where the accuracy of the frequency meters considered here is of the order of 0.1 or 0.2 per cent, and interpolation is by estimation of fractions of a per cent, the use of the interpolating standard improves the accuracy of measurement by 100 to 1000 times.

If a frequency standard and frequency-measuring equipment are available, the interpolating frequency standard can be treated as a highly stable heterodyne frequency meter with a greatly expanded scale. By measurement of the output frequency of the interpolating standard (1000 to 1010 kc), all errors are overcome except that of matching the used harmonic of the interpolating standard to the frequency under measurement. It is then necessary, of course, to multiply the measured frequency, or the measured frequency increment, by the number of the used harmonic to obtain the final result.

By use of a 10-kc harmonic generator, a direct-reading system can be set up as indicated by the dashed section of Fig. 4. Harmonics of the 10-kc harmonic generator between 100 and 200 are used with the frequency standard and frequency-measuring equipment. These harmonics cover 1000 to 2000 kc, and each is readily measured against the 10-kc harmonic series of the frequency standard. If the measurement is made at the harmonic having the *same number* as that used to match the frequency under measurement, then the unknown frequency has been divided effectively by an integral simple number. For example, in Fig. 4, if the 10- to 20-Mc output is used, the frequency under measurement is divided by 10; for the 20- to 50-Mc range, it is divided by 20; for the 50- to 100-Mc range, it is divided by 50; and for the 100- to 200-Mc range, it is divided by 100.

If, then, the frequency difference between the used harmonic and the corresponding harmonic of the frequency standard, in cps, is multiplied by 10, 20, 50, or 100, the result is the *frequency increment to be added to the frequency of the used output* harmonic to obtain the value of the unknown frequency. In practice, the interpolation oscillator of the frequency-measuring system could be fitted with $\times 1$, $\times 2$ and $\times 5$ scales, in which case no multiplication is necessary except to move the decimal point.

The interpolating standard described here is a direct and simple design, predicated on reasonable size and cost. It is evident that greater accuracy is feasible through the use of improved crystal-controlled and interpolation oscillators. However, the best accuracy which can be realized with given equipment used as an interpolating standard will be much below that which can be realized when the equipment is utilized as an element of a conventional frequency-measuring system. For such applications, the function of the crystal-controlled oscillator can be served by selected harmonics of the frequency standard itself, for best possible accuracy, and the function of the interpolation oscillator can be served by a combination of selected harmonics of the standard with a stable variable-frequency oscillator.

The ease and rapidity with which a frequency can be matched at any point in the range; the simplicity of zero-beat settings compared with wide-range interpolation; the effectiveness of the used-harmonic output frequency as a substitute source; and the fact that an output voltage can be generated at any desired frequency in the range, are all factors pointing to the utility of the method.