# Frequency Stable $LC$ Oscillators[*]

J. K. CLAPP†, FELLOW, IRE

*Summary*—A simple theory of the conditions for oscillation, and of the frequency stability of inductance-capacitance oscillators is evolved from a survey of a number of papers on this subject. As some of these papers appeared in publications which are not readily accessible, some of the material may be new to workers in the United States. The condition for oscillation is shown to depend only upon the mutual conductance of the tube and the impedances, tapped on the tuned circuit, presented to the grid and plate circuits of the tube. For linear operation, the stability depends only on the $Q$ of the controlling circuit, and the ratio of the change of interelectrode capacitance to mutual conductance of the tube, and is independent of the $LC$ ratio. For nonlinear operation, however, the stability depends upon the factors given above and on the $LC$ ratio, being improved when a high $LC$ ratio is used. The best tube for high stability is shown to be the tube having the lowest ratio of interelectrode capacitance change to mutual conductance. For highest possible stability, very low level operation with some form of automatic level control is required. A brief historical chronology is included.

THIS DISCUSSION will be limited to that class of oscillator circuits in which the input and output circuits of the tube are connected across portions of the tuned circuit.

It can be shown[1,2] that the condition for oscillation is given by:

$$1/g_m = \sqrt{Z_1' Z_2'} \qquad (1)$$

where $Z_1'$ is the impedance presented by the tapped portion of the tuned circuit to the grid circuit of the tube, and $Z_2'$ is the impedance presented by the tapped portion of the tuned circuit to the plate circuit of the tube. The internal impedances $Z_1$ and $Z_2$, of the input

[1] Jiri Vackar, "$LC$ oscillators and their frequency stability," pp. 1–9, *Telsa Tech. Reports*, Czechoslovakia; December, 1949.
[2] J. K. Clapp, "An inductance-capacitance oscillator of unusual frequency stability," PROC. I.R.E., vol. 36, pp. 356–358; March, 1948; Discussion, W. A. Roberts, PROC. I.R.E., vol. 36, pp. 1261–1262; October, 1948.

and output circuits of the tube are assumed to be large by comparison to the tapped impedances $Z_1'$ and $Z_2'$, which is generally the case in practice.

The effect of a change $\Delta C_1$ in the input capacitance of the tube, connected across the tapped impedance $Z_1'$ of the tuned circuit, causes a detuning equivalent to a change $\Delta C_0$ in the tuned circuit capacitance $C_0$, such that:

$$\Delta C_0/\Delta C_1 = Z_1'/R_0 \tag{2}$$

where $R_0$ is the parallel-resonant impedance of the tuned circuit.

If the tuned-circuit capacitance is $C_0$, a change of $\Delta C_0$ in this capacitance causes a fractional frequency change of

$$\Delta f/f = \Delta C_0/2C_0. \tag{3}$$

Substituting from (2) we have:

$$\Delta f/f = (Z_1'/2R_0)(\Delta C_1/C_0). \tag{4}$$

The larger the impedance $Z_1'$, the larger the frequency change. Similar considerations apply for changes in tube-output capacitance, $\Delta C_2$, and impedance $Z_2'$. For equal changes in *either* grid or plate capacitance, the minimum frequency change, when $Z_1'Z_2'$ is given by (1), occurs when $Z_1' = Z_2' = 1/g_m$ and, since $R_0 = Q/\omega C_0$, is

$$\Delta f/f = (1/2R_0C_0)(\Delta C_1/g_m) = (\omega/2Q)(\Delta C_1/g_m)$$
$$= 2\pi(f/2Q)(\Delta C_1/g_m) = \{1/2(L/R)\}(\Delta C_1/g_m). \tag{5}$$

This condition makes the grid and plate voltages equal, and the tube consequently operates at low efficiency, which is not of prime importance for oscillators where frequency stability is the principal consideration.

In practice, however, it is frequently found that changes in plate-circuit capacitance of the tube are appreciably less than changes in the grid-circuit capacitance. Under such conditions, improved frequency stability and better efficiency can be obtained by not making $Z_1' = Z_2'$.

We can write the total frequency change, caused by changes in *both* grid and plate capacitances as:

$$\Delta f/f = (1/2R_0C_0)(Z_1'\Delta C_1 + Z_2'\Delta C_2). \tag{6}$$

Let $\Delta C_2 = \Delta C_1/k$, then

$$\Delta f/f = (1/2R_0C_0)(Z_1'\Delta C_1 + Z_2'\Delta C_1/k). \tag{7}$$

Remembering that the condition for oscillation requires the product of $Z_1'Z_2'$ to remain constant, divide $Z_1'$ by a factor, $m$, and multiply $Z_2'$ by the same factor. Then

$$\Delta f/f = (1/2R_0C_0)\{(Z_1'/m)\Delta C_1 + mZ_2'(\Delta C_1/k)\} \tag{8}$$

which will be a minimum when the two terms in the right-hand brackets are equal. The original condition called for $Z_1' = Z_2' = 1/gm$.

So we have

$$1/m = m/k, \tag{9}$$

from which

$$m = \sqrt{k} \tag{10}$$

and

$$\Delta f/f = (1/2R_0C_0)(\Delta C_1/gm)(1/\sqrt{k} + 1/\sqrt{k}) \tag{11}$$

for the minimum value.[3] In effect this makes equal the contributions of the grid and plate-circuit capacitances to the total frequency change.

Since $R_0 = Q/\omega C_0$ we can write (11) as

$$\Delta f/f = (\omega/2Q)(\Delta C_1/g_m)(2/\sqrt{k})$$
$$= 2\pi(f/2Q)(\Delta C_1/g_m)(2/\sqrt{k})$$
$$= \{1/2(L/R)\}(\Delta C_1/g_m)(2/\sqrt{k}) \tag{12}$$

when $\Delta C_2 = \Delta C_1/k$.

Equation (12) is instructive since it gives the value of the frequency coefficient immediately, when the quality of the controlling circuit and the $\Delta C_1/g_m$ ratio of the tube are known. If $\Delta C_1$ were independent of $g_m$, that tube having the greatest $g_m$ would give the best frequency stability, and this conclusion has been reached by several writers. In practice, however, the tubes having the larger values of mutual conductance have also the larger values of $C_1$ and larger values of $\Delta C_1$. The choice of a tube having very small tube capacitances, and small capacitance changes, associated with a moderate value of $g_m$ will frequently result in a substantially lower $\Delta C_1/g_m$ ratio and better frequency stability. This is particularly true of secondary changes in tube capacitances such as those caused by changes in heater temperature, for example. Equation (12) also indicates that the frequency coefficient is independent of the $LC_0$ ratio of the tuned circuit, which is true as long as the assumption of linear operation is valid,—a conclusion reached by several writers. However, with nonlinear operation, the frequency coefficient is not independent of the $LC$ ratio, as will be shown later. Equation (12) also states that the stability depends only on the quality of the tuned circuit, and, for a given value of $\Delta C_1$, on the $g_m$ of the tube. This latter term expresses, in effect, the minimum degree of coupling which can exist between the driving circuit and the controlling circuit.

A comparison of a few of a number of circuits which have been developed for frequency stable oscillators is of interest. The circuit, independently developed by Gouriet[4] and Clapp,[2] is probably the simplest and is shown schematically in Fig. 1.

For the impedance presented to the grid circuit of the tube, we have

$$Z_1' = R_0C_v'^2/(C_v'+C_1)^2 \text{ where } C_v' = C_v(1/(1+C_v/C_2)) \tag{13}$$

[3] With $\Delta C_2 = \Delta C_1/10$ and the original condition that $Z_1' = Z_2'$, $\Delta f/f = (\Delta C_1/2R_0C_0\ gm)(1.1)$ from (5). Using (11), $\Delta f/f = (\Delta C_1/2R_0C_0\ gm)(0.632)$, a change which is only about one-half as large.
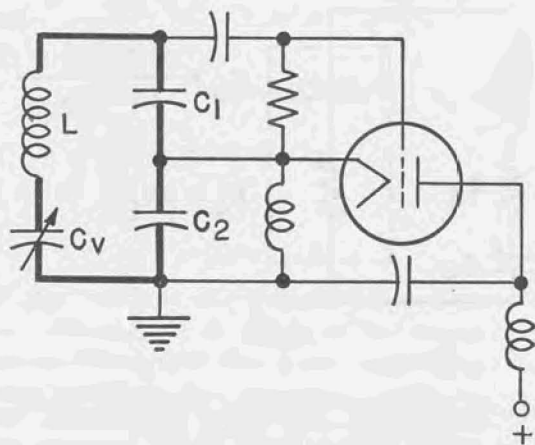[4] G. G. Gouriet, "High stability oscillator," *Wireless Engineer*, pp. 105–112; April, 1950.

Fig. 1—Gouriet-Clapp.



Fig. 2—Seiler.

and, since $C_2 \gg C_v$ and $C_1 \gg C_v$

$$Z_1' = R_0 C_v^2/(C_v + C_1)^2 \cong R_0(C_v/C_1)^2. \qquad (14)$$

From similar considerations,

$$Z_2' \cong R_0(C_v/C_2)^2 \qquad (15)$$

and, for $C_1 = C_2 = C$

$$\sqrt{Z_1'Z_2'} = 1/g_m = R_0(C_v/C)^2 \qquad (16)$$

whence

$$g_m = (1/R_0)(C/C_v)^2 = \omega C^2/Q C_v. \qquad (17)$$

We can obtain a qualitative indication of the change of amplitude with tuning as follows:

$$\omega^2 \cong 1/LC_v \text{ from which } C_v \alpha k/\omega^2. \qquad (18)$$

So

$$g_m \alpha \omega C^2/(Qk/\omega^2) \alpha k\omega^3/Q. \qquad (19)$$

This states that, assuming constant $Q$, the required value of $g_m$ to maintain oscillation increases as the cube of the tuning frequency. In practice this means that as the circuit is tuned to higher frequencies, the amplitude of oscillation will fall and finally the circuit stops oscillating. Even if $Q$ rises somewhat with frequency, as is often true, the falling off in amplitude is still very pronounced.

This oscillator is simple and is useful over a range of about 1.2:1 in frequency, where stability is important.[5]

A parallel counterpart of the Gouriet-Clapp oscillator was described by Seiler.[6] The circuit is given schematically in Fig. 2.

For the impedance presented to the grid circuit

$$Z_1' = R_0 X_1^2/(X_1 + X_z')^2 \cong R_0 C_x^2/(C_1 + C_x)^2$$
$$\cong R_0(C_x/C_1)^2 \qquad (20)$$

with similar considerations for $Z_2'$.

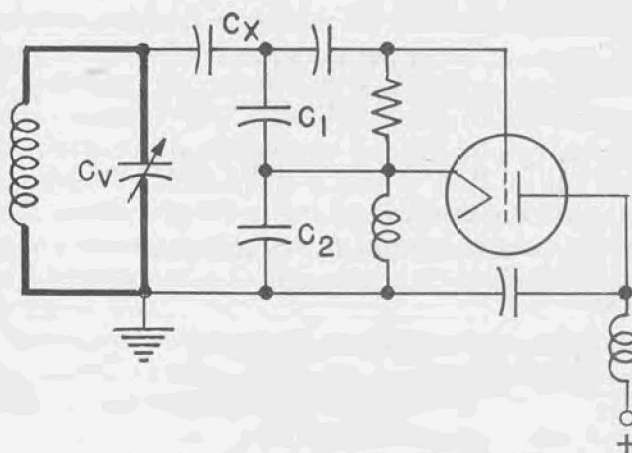Then

$$1/g_m = \sqrt{Z_1'Z_2'} = R_0(C_x/C)^2 \qquad (21)$$

$$g_m = (1/R_0)(C/C_x)^2 = (\omega C_v/Q)(C/C_x)^2. \qquad (22)$$

Now

$$C_v \alpha k/\omega^2. \qquad (23)$$

So

$$g_m \alpha k/\omega Q. \qquad (24)$$

With this circuit, assuming constant $Q$, the $g_m$ required for oscillation is proportional to $1/\omega$, so that as the tuning is changed toward higher frequencies, the amplitude rises. This would be increased, if, as is often true, $Q$ increases with frequency. This oscillator is useful over frequency ranges of about 1.8:1.

The inductive counterpart of Seiler's circuit, described by Lampkin,[7] operates in the same manner. The tube is connected to points tapped on the inductive branch of the tuned circuit. The circuit shows a rather strong tendency to break into spurious oscillation, because of the inductive reactances across the tube input and output circuits.

Vackar[1] describes a circuit combining the features of the series and parallel arrangements and it is shown schematically in Fig. 3, on the following page.

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[5] In the author's paper[2] describing this circuit, the condition for oscillation was expressed in terms of the *series* impedance of the tuned circuit as:

$$g_m X_1 X_2 + X_1^2/r_g + X_2^2/r_p = R_s \qquad (a)$$

which, for practical cases, reduces to:

$$g_m X_1 X_2 = R_s \qquad (b)$$

now express $R_s$ in terms of the *parallel* resonant impedance, $R_0$, of the tuned circuit, by writing $X_0^2/R_0$ for $R_s$:

$$g_m X_1 X_2 = X_0^2/R_0 \qquad (c)$$

or

$$1/g_m = R_0(X_1 X_2/X_0^2)$$
$$= \sqrt{R_0^2(X_1/X_0)^2(X_2/X_0)^2} = \sqrt{Z_1'Z_2'} \qquad (d)$$

and, if $X_1 = X_2 = X$

$$1/g_m = R_0(C_0/C)^2 \cong R_0(C_v/C)^2 \qquad (e)$$

which is in the form given by Vackar.

[6] E. O. Seiler, "A variable frequency oscillator," *QST*, pp. 26–27; November, 1941.

[7] G. F. Lampkin, "An improvement in constant frequency oscillators," Proc. I.R.E., vol. 27, pp. 199–201; March, 1939.

Here

$$C_v' = C_v + C_x C_1/(C_x + C_1) \tag{25}$$
$$\cong C_v \text{ when } C_1 \gg C_x \text{ and } C_v \gg C_x \tag{26}$$

$$Z_1' = R_0\{C_2{}^2/(C_v' + C_2)^2\}\{C_x{}^2/(C_x + C_1)^2\}$$
$$\cong R_0(C_x/C_1)^2 \tag{27}$$
$$Z_2' = R_0 C_v'^2/(C_v' + C_2)^2 \cong R_0(C_v/C_2)^2. \tag{28}$$

Then

$$g_m = (1/R_0)(C_2/C_v)(C_1/C_x)$$
$$= (\omega C_v/Q)(C_2/C_v)(C_1/C_x) \tag{29}$$
$$= (\omega/Q)(C_1 C_2/C_x).$$

If $Q$ is constant, the $g_m$ required to maintain oscillation rises with the frequency, so the amplitude would slowly fall.
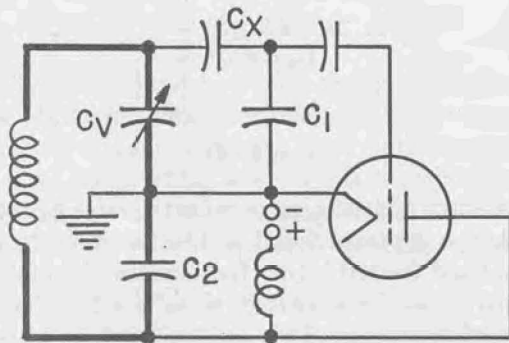


Fig. 3—Vackar.

If $Q$ increases with frequency, however, the amplitude tends to remain reasonably constant. This circuit is useful over frequency ranges as great as 2.5:1.

Vackar,[1] Gouriet[4] and Edson[8] point out that under the condition of linear operation the stability is independent of the $LC_v$ ratio. If this ratio is made zero, the "series-tuned" oscillator, of Fig. 1, becomes simply a Colpitts oscillator. To realize the correct impedance values to be presented to the tube, in order to maintain the frequency stability, the circuit reactances of a simple Colpitts oscillator become impracticably small.[4,8]

There is an important cause for frequency instability, which is wholly neglected in the linear theory, and that is the effect of harmonic components due to the distortion caused by the tube. Llewellyn[9] has shown that, by intermodulation, the harmonic components can cause a phase shift at the fundamental frequency. This phase shift can be considered as an equivalent modification of the generator impedance.[4] This modification can be accounted for as a change in the generator capacitance, $C_g$, since the real part of the generator impedance must equal the loss resistance of the tuned circuit, which has been assumed to be constant.

[8] W. A. Edson, "Vacuum Tube Oscillators," John Wiley and Sons, Inc., New York, N. Y., pp. 170–172; 1953.
[9] F. W. Llewellyn, "Constant frequency oscillators," Proc. I.R.E., vol. 19, pp. 2063–2094; December, 1931.
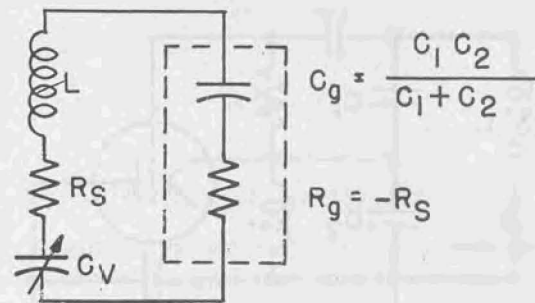


Fig. 4

For the discussion of the effect of distortion, it is convenient to reduce the schematic of Fig. 1, to the equivalent of Fig. 4. The change in frequency, resulting from a change in generator capacitance, $C_g$, is found as follows:

The generator phase angle is

$$\phi = \tan^{-1} 1/\omega C_g R_g \tag{30}$$

$$\frac{dC_g}{d\phi} = -(\omega^2 R_g{}^2 C_g{}^2 + 1)(1/\omega R_g)$$

$$= -(1/\omega R_g + \omega R_g C_g{}^2)$$

$$= 1/\omega R_s + \omega R_s C_g{}^2 \text{ since } R_g = -R_s. \tag{31}$$

The frequency is

$$f = 1/2\pi\sqrt{LC_v C_g/(C_v + C_g)}$$

$$= f_0\sqrt{1 + C_v/C_g} \text{ where } f_0 = 1/2\pi\sqrt{LC_v} \tag{32}$$

and the change in frequency, with change of $C_g$ is

$$\frac{df}{dC_g} = -(f_0/2\sqrt{1 + C_v/C_g})(C_v/C_g{}^2)$$

$$= -(f_0/2)(C_v/C_g)^2 \text{ since } C_g \gg C_v. \tag{33}$$

From the condition for oscillation, when $C_1 = C_2 = 2C_g$

$$\omega^2 C_g{}^2 = g_m/4R_s \tag{34}$$

and

$$C_g{}^2 = g_m/4\omega^2 R_s. \tag{35}$$

Then

$$\frac{dC_g}{d\phi} = 1/\omega R_s + g_m/4\omega. \tag{36}$$

Substitute $1/\omega C_v Q$ for $R_s$ in (35) obtaining

$$C_g{}^2 = g_m C_v Q/4\omega \tag{37}$$

from which

$$C_v/C_g{}^2 = 4\omega/g_m Q. \tag{38}$$

Substitute in the expression (33) obtaining:

$$\frac{df}{dC_g} = -f_0 C_v/2C_g{}^2 = -4\pi f_0{}^2/g_m Q. \tag{39}$$

Then

$$\frac{df}{d\phi} = \frac{df}{dC_g} \cdot \frac{dC_g}{d\phi} = - (4\pi f_0^2/g_m Q)(1/\omega R_s + \omega R_g C_g^2)$$

$$= - (f/2Q + 1/\pi g_m L), \tag{40}$$

since

$$Q = \omega L/R_s.$$

The first term is the differential coefficient of frequency with respect to phase of the tuned circuit at resonance. The second term is very much larger than the first, and indicates that increasing $L$ will reduce its effect. In other words, when distortion is present, a circuit of high $LC_g$ ratio is desirable for best stability, whereas in the linear case the stability is independent of the $LC_g$ ratio.

The effect of a small quadrature current flowing through the generator impedance could produce a relatively large frequency change, which would be quite sensitive to changes in plate supply voltage, for example. Such a quadrature current might be caused by unintentional feedback from a subsequent amplifier stage. The use of a high $LC_g$ ratio in the tuned circuit can reduce the frequency change caused by phase change by 100 or more times over the change experienced in a simple Colpitts oscillator.

All of the above brings out the fact that careful connection of output amplifiers is necessary, and that the tube must be operated in as nearly a linear manner as possible. Taking the output across a low resistance in the plate circuit and using some form of automatic level control are proper steps.
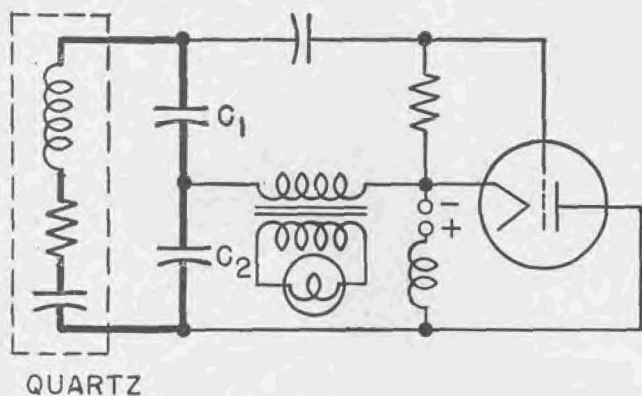


QUARTZ

Fig. 5—Hansen.

The "series-tuned" circuit stems from the crystal oscillator which we have used for several years. The shunt capacitors assume lower values than in $LC$ circuits because of the extremely small series capacitance of the quartz vibrator. The resistance of the quartz vibrator is also much higher than that of an $LC$-tuned circuit.

Hansen[10] describes a crystal oscillator of this type with a lamp in the cathode lead to provide automatic control of amplitude, Fig. 5. To adjust the effective

value of feedback resistance to the desired value, the lamp is coupled through a transformer.

Analysis of the circuit, with feedback, results in equations identical with those obtained with no feedback except that $g_m$ is replaced by $g_m'$, the reduced value of $g_m$ caused by feedback. If a lamp is used for the feedback resistor, the effective resistance becomes a function of ac-plate current, so that an increase in level is offset by a reduction of $g_m'$. This control is obtained without change of bias.

Enhanced control could be obtained by amplifying the oscillator output, rectifying it and applying the rectified current to the lamp.
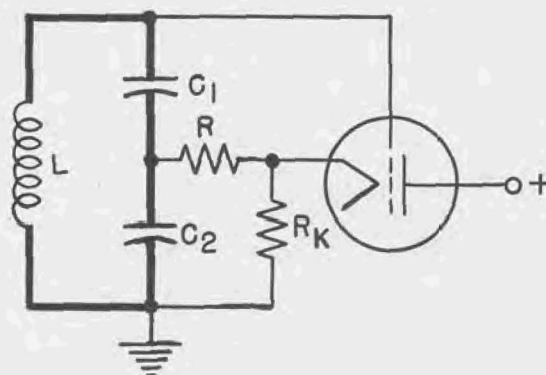


Fig. 6—Harris.

A circuit somewhat similar to the above feedback circuit has been described by Harris[11] as a "Q multiplier" circuit, Fig. 6. In this circuit, a cathode follower amplifier is connected through a high resistance to a tap on a tuned circuit, the high impedance point of which is returned to the grid. If the drop in output voltage to the tap on the tuned circuit is offset by the voltage stepup of the tuned circuit, the circuit will oscillate. If the gain of the completely degenerated amplifier approximates unity, then the value of series resistance is $R_0/4$ for oscillation, if the tuned circuit is tapped halfway up. In this oscillator, the output circuit is almost completely isolated from the tube output circuit; the tube input circuit is placed across the entire tuned circuit. Since changes in tube input capacitance are reduced by feedback, this circuit has possibilities as a stable oscillator, particularly for low frequencies.

## HISTORICAL CHRONOLOGY

The criterion for oscillation, $1/g_m = \sqrt{Z_1' Z_2'}$, means that the highest stability, with respect to changes of the internal capacitances of the tube, can be achieved by connecting the grid and plate circuits to points on the tuned circuit of as low impedance as possible and still maintain oscillation.

This criterion, expressed in slightly different ways, was discovered by a number of authors (as mentioned in this paper) and was realized in various forms of circuits.

[10] H. N. Hansen, "A crystal oscillator for carrier supply," *Philips Tele. News*, vol. X, pp. 1–15; January, 1949.

[11] H. E. Harris, "A simplified Q multiplier," *Electronics*, pp. 130–134; May, 1951.

The oscillator developed by Gouriet, which it is stated, has been used in the B. B. C. since 1938,[4] was not described in the technical press until 1947 and then in a book, "Radio Engineering" by E. K. Sandeman. The circuit was independently developed by Clapp in 1946 (described in the PROCEEDINGS OF THE I.R.E., 1948).[2] The circuits developed by Seiler (*QST*, 1941)[6] and Lampkin (PROCEEDINGS OF THE I.R.E., 1939)[7] follow the same criterion, but were not described clearly on the impedance concept.

During the war development of stable oscillators in Czechoslovakia was carried out independently and without exchange of technical information with the West. The circuit of Fig. 3 of this paper was developed by *Radioslavia* in 1945, but publication did not occur until 1949.[1] Meanwhile, the same circuit was developed independently by O. Landini in Italy and was described in *Radio Rivista*, 1948.

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